5.4 A Continuous-Flow Stirred –Tank Reactor with Effluent Recycle

• CSTR: the same as chemostat
  - continuous flow in and out
  - reach a steady state, VdC/dt =0

• CSTR with effluent recycle (Fig 5.4)
  - some of the effluent stream is recycled back at a flow rate Qr.

How does “effluent recycle” affect reactor performance?
  - It does not affect reactor performance at all.
  - Which *(the entire system or the reactor itself)* is used as the control volume makes no difference to the results.
  - But, CSTR with effluent recycle can increase the total reactor volume by the volume of recycle line and increase the extent of mixing.
Mass Balance

\[ QS^0 + Q^r S = Q^i S^i \quad \text{and} \quad QXa^0 + Q^r Xa^r = Q^i Xa^i \]

\[ S^i = \frac{QS^0 + Q^r S}{Q^i} \quad [5.12] \]

\[ Q^i = Q + Q^r \quad [5.13] \]

for the steady-state case

\[ 0 = Q^i S^i - Q^i S + r_{ut} V \quad [5.14] \]
5.4 A Continuous-Flow Stirred –Tank Reactor with Effluent Recycle

substitutions from Equations 5.12 and 5.13

\[ 0 = Q(S^0 - S) + r_{ut} V \]  \[5.15\]

• Equation 5.15 is identical to Equation 3.15, the case for chemostat without recycle.

\[ 0 = r_{ut} V + Q(S^0 - S) \]  \[3.15\]

• Simple recycle for a CSTR does not change substrate removal compared with that obtained without recycle.

• Organism concentrations within the reactor and in the reactor effluent are not affected by effluent recycle, since the same mass flow that leaves the reactor returns to the reactor.
5.5 A Plug-Flow Reactor

- PFR
  - the substrate and active-organism concentrations vary over the length of the reactor.

  - Substrate
    \[
    \Delta V \frac{\Delta S}{\Delta t} = QS - Q(S + \Delta S) + r_{ut} \Delta V \tag{5.16}
    \]

  - Active microorganisms
    \[
    \Delta V \frac{\Delta Xa}{\Delta t} = QXa - Q(Xa + \Delta Xa) + r_{net} \Delta V \tag{5.17}
    \]

  \(r_{ut}\) and \(r_{net}\) are the reaction rates for substrate (Eq. 3.6) and active organisms (Eq. 3.8).
5.5 A Plug-Flow Reactor

- At steady-state case, for which influent flow rate (Q), substrate concentration (S) and active organism concentration (Xa) do not change with time, the left sides of Eq. 5.16 and 5.17 are zero.

\[
0 = QS - Q(S + \Delta S) + r_{ut} \Delta V
\]

\[
0 = QXa - Q(Xa + \Delta Xa) + r_{net} \Delta V
\]

- Area of the control volume (A): \( A = \Delta V / \Delta z \)

- Velocity of flow within the reactor (u): \( u = Q/A = Q/ (\Delta V / \Delta z) = Q \times \Delta z / \Delta V \)
5.5 A Plug-Flow Reactor

- Substrate at steady-state

\[ 0 = QS - Q(S + \Delta S) + r_{ut} \Delta V \]
\[ u = Q \times \Delta z / \Delta V \]

\[ u \frac{\Delta S}{\Delta z} = r_{ut} \] [5.18]

- Active microorganisms at steady-state

\[ 0 = QXa - Q(Xa + \Delta Xa) + r_{net} \Delta V \]
\[ u = Q \times \Delta z / \Delta V \]

\[ u \frac{\Delta Xa}{\Delta z} = r_{net} \] [5.19]

- Assumption: 1) \( \Delta z \) approach zero

2) Monod reaction applies for substrate utilization (Eq. 3.6)

3) Organisms net growth represents growth and decay (Eq. 3.8)

\[ u \frac{dS}{dz} = -\hat{q} \frac{S}{K + S} Xa \] [5.20]

\[ u \frac{dXa}{dz} = \hat{Y}q \frac{S}{K + S} Xa - bXa \] [5.21]
5.5 A Plug-Flow Reactor

- If we ignore organism decay (b=0) and combine Eq. 5.20 and Eq. 5.21, then analytical solution is possible.

\[ u \frac{dXa}{dz} = -uY \frac{dS}{dz} \quad [5.22] \]

\[ \int_{Xa0}^{Xa} dXa = -Y \int_{So}^{S} dS \quad [5.23] \]

\[ Xa = Xa^0 + Y(S^0 - S) \quad [5.24] \]

- Substituting Eq. 5.24 into Eq. 5.20 gives a differential eq with only two variables, s and z.

\[ u \frac{dS}{dz} = -\hat{q} \frac{S}{K + S} Xa \quad [5.20] \]

\[ u \frac{dS}{dz} = -\hat{q} \frac{S}{K + S} [Xa^0 + Y(S^0 - S)] \quad [5.25] \]
5.5 A Plug-Flow Reactor

- The ratio $\frac{dz}{u} = dt$ (the time for an element of water to move a distance $dZ$)

- Substituting $dt$ for $\frac{dz}{u}$ in Eq 5.25 yields for a differential equation that is the same as Eq. 5.10 for batch reactor.

$$u \frac{dS}{dz} = -\hat{q} \frac{S}{K + S} \left[ Xa^0 + Y(S^0 - S) \right] \quad [5.25]$$

$$\frac{dS}{dt} = -\frac{\hat{q}S}{K + S} \left[ Xa^0 + Y(S^0 - S) \right] \quad [5.10]$$

- Integration of eq 5.25 gives eq 5.26
  which is almost identical to eq 5.11 for a batch reactor.

$$\frac{z}{u} = \frac{1}{\hat{q}} \left\{ \left( \frac{K}{Xa^0 + YS^0} + \frac{1}{Y} \right) \ln \left\{ Xa^0 + YS^0 - YS \right\} - \left( \frac{K}{Xa^0 + YS^0} \right) \ln \frac{SXa^0}{S^0} - \frac{1}{Y} \ln Xa^0 \right\} \quad [5.26]$$

$$t = \frac{1}{\hat{q}} \left\{ \left( \frac{K}{Xa^0 + YS^0} + \frac{1}{Y} \right) \ln \left( Xa^0 + YS^0 - YS \right) - \left( \frac{K}{Xa^0 + YS^0} \right) \ln \frac{SXa^0}{S^0} - \frac{1}{Y} \ln Xa^0 \right\} \quad [5.11]$$
5.5 A Plug-Flow Reactor

\[
\frac{z}{u} = \frac{1}{\hat{q}} \left\{ \left( \frac{K}{Xa^0 + YS^0} + \frac{1}{Y} \right) \ln \{ Xa^0 + YS^0 - YS \} - \left( \frac{K}{Xa^0 + YS^0} \right) \ln \frac{S Xa^0}{S^0} - \frac{1}{Y} \ln Xa^0 \right\}
\]

[5.26]

- An expression for the effluent concentration (S) from the batch reactor is obtained by letting \( z = L \).

\[
\frac{L}{u} = \frac{1}{\hat{q}} \left\{ \left( \frac{K}{Xa^0 + YS^0} + \frac{1}{Y} \right) \ln \{ Xa^0 + YS^0 - YS \} - \left( \frac{K}{Xa^0 + YS^0} \right) \ln \frac{S Xa^0}{S^0} - \frac{1}{Y} \ln Xa^0 \right\}
\]

\[L/u = \frac{V}{Q} = \theta \text{ (the hydraulic detention time)}\]

\[
\theta = \frac{1}{\hat{q}} \left\{ \left( \frac{K}{Xa^0 + YS^0} + \frac{1}{Y} \right) \ln \{ Xa^0 + YS^0 - YS^e \} - \left( \frac{K}{Xa^0 + YS^0} \right) \ln \frac{S^e Xa^0}{S^0} - \frac{1}{Y} \ln Xa^0 \right\}
\]

[5.27]

- Eq 5.27 is identical to eq 5.11 with \( \theta \) replacing \( t \).
5.5 A Plug-Flow Reactor

- We thus see that a PFR works exactly like a batch reactor.

- In practice, however, it is difficult to operate a PFR according to the assumptions involved.

   1) A PFR has no mixing or short-circuiting of the fluid along the flow direction. This is impossible to achieve in a real continuous-flow reactor.

   2) Wall effects slow the fluid near the wall boundaries relative to the velocity near the middle.

   3) Aeration or mixing to keep the biomass in suspension introduces a large amount of mixing in all directions.
5.5 A Plug-Flow Reactor

- Methods to achieve as much of a plug-flow character as possible include
  1) using a very long, narrow reactor and 2) using many reactors in series.
  But, some mixing and short-circuiting are inevitable.

- If achieving the reaction kinetics represented by eq 5.27 is of paramount importance, a batch reactor is prudent choice (e.g., choice of batch reactor instead of PFR might be better), although it presents its own problems:

  For example, time is required to fill and empty a batch reactor, time that might otherwise be used for treatment. In order to minimize downtime (e.g., idle time), a batch reactor can be operated while it is filling.

\[
\theta = \frac{1}{\hat{q}} \left\{ \left( \frac{K}{Xa^0 + YS^0} + \frac{1}{Y} \right) \ln \left\{ Xa^0 + YS^0 - YS^e \right\} - \left( \frac{K}{Xa^0 + YS^0} \right) \ln \frac{S^e Xa^0}{S^0} - \frac{1}{Y} \ln Xa^0 \right\}
\]

[5.27]
5.6 A Plug-Flow Reactor with Effluent Recycle

- If no organisms are introduced in PFR, then the system fails to do any treatment.
- Using effluent recycle, a portion of microorganisms in the effluent is brought back to the influent stream.

\[
S^i = \frac{Q_S^0 + Q^r S}{Q^i} \quad \text{and} \quad X^i_a = \frac{Q X_a^0 + Q^r X^r_a}{Q^i} \quad [5.12]
\]

\[
Q^i = Q + Q^r \quad [5.13] \quad X^r = X^e \quad \text{and} \quad S = S^e
\]
5.6 A Plug-Flow Reactor with Effluent Recycle

- The reaction occurring are same manner as with the simple PFR except that,

\[ Q^0, X^0 \text{ and } S^0 \rightarrow Q^i, X^i \text{ and } S^i \]

- Ignore organism decay \((b=0)\), eq 5.28. And 5.29 can be obtained from Eq. 5.24 and 5.26

\[ X_a = X_a^0 + Y(S^0 - S) \quad [5.24] \]

\[ \frac{z}{u} = \frac{1}{\hat{q}} \left\{ \left( \frac{K}{X_a^0 + YS^0} + \frac{1}{Y} \right) \ln \{ X_a^0 + YS^0 - YS \} - \left( \frac{K}{X_a^0 + YS^0} \right) \ln \left( \frac{SXa^0}{S^0} \right) - \frac{1}{Y} \ln X_a^0 \right\} \quad [5.26] \]

\[ X_a^e = X_a^0 + Y(S^0 - S^e) \quad [5.28] \]

\[ \frac{V}{Q^i} = \frac{1}{\hat{q}} \left\{ \left( \frac{K}{X_a^i + YS^i} + \frac{1}{Y} \right) \ln (X_a^i + YS^i - YS^e) - \left( \frac{K}{X_a^i + YS^i} \right) \ln \left( \frac{S^e X_a^i}{S^i} \right) - \frac{1}{Y} \ln X_a^i \right\} \quad [5.29] \]
5.6 A Plug-Flow Reactor with Effluent Recycle

\[
\frac{V}{Q^i} = \frac{1}{q} \left\{ \left( \frac{K}{X^i_a + YS^i} + \frac{1}{Y} \right) \ln (X^i_a + YS^i - YS^e) - \left( \frac{K}{X^i_a + YS^i} \right) \ln \frac{S^e X^i_a}{S^i} - \frac{1}{Y} \ln X^i_a \right\} \quad [5.29]
\]

- Recycle ratio \( R \),
  \[
  R = \frac{Q^r}{Q} \quad [5.30]
  \]

- Detention time \( \theta \),
  \[
  \theta = \frac{V}{Q_i} = \frac{V(1 + R)}{Q^i} \quad [5.31]
  \]
5.6 A Plug-Flow Reactor with Effluent Recycle

- Graph with $S^e$, $\theta$, $R$
  - Solve equation using spreadsheet to determine $S^e$ as a function of $\theta$ and $R$
    - Upper graph; arithmetic scale
    - Lower; log scale
- The washout detention time is larger when $R$ decreases

- For the same condition, the $\theta_{min}$ for the CSTR is 0.2 days (close to the value for $R=8$ for plug-flow with recycle).

- In theory, when $R$ approaches infinity, the PFR with recycle becomes identical to a CSTR.
5.6 A Plug-Flow Reactor with Effluent Recycle

Figure 5.7: Effect of recycle ratio ($R$) and hydraulic detention time ($\theta$) on effluent substrate concentration for a plug-flow reactor with effluent recycle. Case considered has $S^0 = 100$ mg/l, $Y = 0.6$ mg/mg, $K = 20$ mg/l, $q = 10$ mg/mg-d, and $b = 0$. 
5.6 A Plug-Flow Reactor with Effluent Recycle

- Relative benefits of a CSTR and a plug-flow reactor with recycle

  - R=8 is very similar to a CSTR ‘s performance
  ① - CSTR improves reliability if contaminant removal of 80 to 90 % were satisfactory.

  ② - PFR with low recycle ratio is much more desirable if contaminant removal of 99.9% were required.

  - There is an optimal recycle ratio that provide low θ and high efficient contaminant removal.

- Effluent recycle with a CSTR does not change system performance, but with a PFR, recycle is essential and has a great impact on performance.
5.7 Reactors with **Recycle of Settled Cells**

- **Microorganism recycle from a settling tank**: the most widely used suspended-growth reactor.

- **Any method that increases the con. of microorganisms in the reactor increases the volumetric reaction rate and, in this manner, decreases the required reactor volume**

- The **primary advantage**: a much smaller reactor volume is required.

  The **disadvantage**: the cost of the settler and the recycling system.

  **Smaller reactor plus settler vs a larger reactor without a settler**
5.7.1 CSTR with Settling and Cell Recycling

- Assumptions
- Mass balance for microorganisms
- Mass balance for substrates
- Solids retention time (SRT)

CSTR with Settling and Cell Recycling

Applicable Equations: $S, X_a$
5.7.1 CSTR with Settling and Cell Recycling

Assumptions:

- Biodegradation of the substrates takes place in the reactor only, no biological reactions take place in the settling tank, and biomass in the settler is insignificant.
- No active microorganisms are in the influent to the reactor ($X_a^0 = 0$).
- The substrate is soluble so that it can not settle out in the settling tank.
5.7.1 CSTR with Settling and Cell Recycling

Accumulation = In – Out + Generation - Consumption

Mass balance for microorganism:

\[ V \frac{dX_a}{dt} = 0 - (Q^e X^e_a + Q^w X^w_a) + [Y (-r_{ut}) V - bX_a V] \]  \[ 5.32 \]

Mass balance for substrates:

\[ V \frac{dS}{dt} = Q^0 S^0 - (Q^e S^e + Q^w S^w) + r_{ut} V \]  \[ 5.33 \]
5.7.1 CSTR with Settling and Cell Recycling

\[ \theta_x = \frac{\text{active biomass in the system}}{\text{production rate of active biomass}} \]

At steady state,

\[ \theta_x = \frac{\text{active biomass in the system}}{\text{removal rate of active biomass}} = \frac{X_a V}{Q^e X_a^e + Q^w X_a^w} \quad [5.35] \]
At steady-state, from eq 5.32

\[ 0 = 0 - (Q^e X_a^e + Q^w X_a^w) + [Y (-r_{ut}) V - bX_a V] \]

\[ \frac{Q^e X_a^e + Q^w X_a^w}{X_a V} = \frac{Y (-r_{ut})}{X_a} - b \]  \[5.36\]

\[ \theta_x = \frac{X_a V}{Q^e X_a^e + Q^w X_a^w} \]  \[5.35\]

Seeing the similarity between the left side of Eq.5.36 and the right side of eq. 5.35

\[ \frac{1}{\theta_x} = \frac{Y (-r_{ut})}{X_a} - b \]  \[5.37\]

If it takes Monod kinetics,

\[ \frac{1}{\theta_x} = \frac{Y (-r_{ut})}{X_a} - b = Y \frac{\hat{q}S}{K + S} - b \]  \[5.38\]
5.7.1 CSTR with Settling and Cell Recycling

\[
\frac{1}{\theta_x} = \frac{Y(-r_{ut})}{X_a} - b = Y \frac{\hat{q}S}{K + S} - b \quad [5.38]
\]

Solving this equation explicitly for \( S \),

\[
S = K \frac{1 + b \theta_x}{\theta_x (Y q - b) - 1} \quad [5.39]
\]

\[
S = K \frac{1 + b \theta_x}{Y \hat{q} \theta_x - (1 + b \theta_x)} \quad [3.24] \text{ where } \theta_x = \theta
\]

Eq 5.39 is identical to eq 3.24, which was developed for the chemostat without settling and recycle.

So then, what is unique about the CSTR with settling and microorganism recycle?
5.7.1 CSTR with Settling and Cell Recycling

So then, what is unique about the CSTR with settling and microorganism recycle?

\[
S = K \left( 1 + b \theta_x \right) \frac{1}{\theta_x (Y q - b) - 1} \quad [5.39]
\]

\[
S = K \frac{1 + b \theta_x}{Y \dot{q} \theta_x - (1 + b \theta_x)} \quad [3.24]
\]

Answer: the retention time of microorganisms in the system ($\theta_x$) is separated from the hydraulic retention time ($\theta$)

- In eq 3.24 without microorganisms recycle, $\theta_x = \theta$
  
  But in eq 5.39 with microorganisms recycle, $\theta_x$ is not necessarily equal to $\theta$

- Thus one can have a large $\theta_x$, in order to obtain high efficiency of substrate removal, and at the same time have a small $\theta$.

For example, $\theta_x = 4-10 \, d$ (small $S$, substrate),

while $\theta = 4-10 \, h$ (small $V$, volume).

Thus, while operating at the same treatment efficiency for substrate, the reactor size can be $1/24$ of the volume of CSTR without settling and recycle.
5.7.1 CSTR with Settling and Cell Recycling

\[
\frac{1}{\theta} = \frac{Y(-r_{ut})}{X_a} - b \quad [5.37]
\]

From eq 5.37,
\[
X_a = \theta \frac{Y(-r_{ut})}{1 + b\theta_x} \quad [5.40]
\]

From Mass balance for substrates (eq 5.33 & 5.34)
\[
0 = Q^0 S^0 - (Q^e S^e + Q^w S^w) + r_{ut} V
\]
\[
-r_{ut} = \frac{Q^0 S^0 - Q^e S^e - Q^w S^w}{V} \quad [5.41]
\]

since no reaction occurs in the settling tan k
\[
S = S^e = S^w, \quad Q^0 = Q^e + Q^w, \text{then}
\]
\[
-r_{ut} = \frac{Q^0 (S^0 - S)}{V} = \frac{(S^0 - S)}{\theta} \quad [5.42]
\]

Substituting eq 5.42 into eq 5.40,
\[
X_a = \theta \frac{Y(-r_{ut})}{1 + b\theta_x} = \frac{\theta}{\theta} \frac{Y(S^0 - S)}{1 + b\theta_x} \quad [5.43]
\]
5.7.1 CSTR with Settling and Cell Recycling

\[ X_a = \frac{\theta_x}{\theta} \frac{Y(S^0 - S)}{1 + b \theta_x} \]  \hspace{1cm} [5.43]

\[ \frac{\theta_x}{\theta} : \text{Solids concentration ratio} \]

- Active biomass concentration in the reactor depends on the ratio of solids retention time to the hydraulic detention time,

- For a CSTR without settling and recycle, \( \frac{\theta_x}{\theta} = 1 \) and then eq 5.43 is equal to eq 3.25.

\[ X_a = Y \left( \frac{S^0 - S}{1 + b \theta_x} \right) \]  \hspace{1cm} [3.25]

- If \( \frac{\theta_x}{\theta} = 24 \), and \( \theta_x \) is same, then \( X_a \) with settling is 24 times higher than it would be without biomass recycling
Mass rate of active biomass production

At steady state, the mass rate of active biomass production must equal the rate at which the biomass leaves the system from the effluent stream and the waste stream.

Combining eq 5.44 and eq 5.35 yields eq. 5.45

\[ r_{abp} : \text{active biomass production} \quad (M / T) \]

\[ \theta_x = \frac{X_a V}{Q^e X_a^e + Q^w X_a^w} \quad [5.35] \]

\[ r_{abp} = Q^e X_a^e + Q^w X_a^w \quad [5.44] \]

\[ = \frac{X_a V}{\theta_x} \quad [5.45] \]
5.7.1 CSTR with Settling and Cell Recycling

Table 5.2 summarizes a series of equations to design a CSTR with settling and recycle.

- Assumptions for Table 5.2
  i) Operating at steady state
  ii) Treating a soluble substrate
  iii) No input of active biomass

- The equations in Table 5.2 can be used for a CSTR without a settler by letting $\theta_x = \theta$. 
5.7.1 CSTR with Settling and Cell Recycling

\[ \theta_x = \theta \]

- **Hydraulic Detention Time**

\[
\theta = \frac{V}{Q^0} \tag{3.20}
\]

- **Solids Retention Time, SRT**

\[
\theta_x = \frac{VX_a}{X_a Q^e + X_a Q^w} \tag{5.35}
\]

\[
\frac{1}{\theta_x} = \frac{Y(-r_{ut})}{X_a} - b = Y \frac{\hat{q}S}{K + S} - b \tag{5.38}
\]
5.7.1 CSTR with Settling and Cell Recycling

- $\theta_{x}^{\text{min}}$: SRT at which microorganism washout results and the limit thereto:

- The minimum value of the mean cell residence time and its limiting value are identical to the case without settling (eq 3.26 & 3.27)

\[
\theta_{x}^{\text{min}} = \frac{K + S^0}{\hat{S}^0 (Y \hat{q} - b) - Kb} \quad S \to S^0 \quad [3.26]
\]

\[
[\theta_{x}^{\text{min}}]_{\text{lim}} = \frac{1}{Y \hat{q} - b} \quad S^0 \to \infty \quad [3.27]
\]
5.7.1 CSTR with Settling and Cell Recycling

- Reactor of Effluent substrate concentration

\[ S = K \frac{1 + b \theta_x}{\theta_x (Y q - b) - 1} \]  \[ \text{[3.24] & [5.39]} \]

- Reactor Minimum Substrate Concentration

\[ S_{\text{lim}} = K \frac{b}{Y \hat{q} - b} \quad \theta_x \to \infty \]  \[ \text{[3.28]} \]

- Reactor active Microorganism Concentration

\[ X_a = \theta_x \frac{Y (-r_{ut})}{1 + b \theta_x} \]  \[ \text{[5.40]} \]

\[ X_a = \frac{\theta_x}{\theta} \frac{Y (S^0 - S)}{1 + b \theta_x} \]  \[ \text{[5.43]} \]
5.7.1 CSTR with Settling and Cell Recycling

- **Reactor active Microorganism Concentration**

\[
X_a = \frac{\theta_x Y (-r_{ut})}{1 + b \theta_x} \quad [5.40]
\]

\[
X_a = \frac{\theta_x Y (S^0 - S)}{1 + b \theta_x} \quad [5.43]
\]

- Because inert biomass and total volatile solids are particles, like active biomass, the concentrations \(X_i\) and \(X_v\) take the same form as for a chemostat, but are multiplied by the solids-concentration factor (eq 5.46 & 5.47).

- **Reactor Inert Microorganism Concentration**

\[
X_i = \frac{\theta_x}{\theta} \left[ X_i^0 + X_a (1 - f_d) b \theta \right] \quad [5.46]
\]

- **Reactor Volatile suspended solids concentration**

\[
X_v = X_i + X_a
\]

\[
X_v = \frac{\theta_x}{\theta} \left[ X_i^0 + \frac{Y (S^0 - S)(1 + (1 - f_d) b \theta_x)}{1 + b \theta_x} \right] \quad [5.47]
\]
5.7.1 CSTR with Settling and Cell Recycling

- **Active Biological Sludge Production Rate**

\[ r_{abp} = \frac{X_a V}{\theta_x} \tag{5.45} \]

- The total biological-solids production rate is analogous to eq 5.45

- **Total Biological Solids Production Rate**

\[ r_{tbp} = \frac{X_r V}{\theta_x} \tag{5.48} \]
5.7.1 CSTR with Settling and Cell Recycling

- At a constant SRT, the effluent conc. \((S)\) remains the same regardless of the influent conc. \((S^0)\). Only \(\theta_x\) affects \(S\) because all other parameters in the equations are coefficients.

\[
S = K \frac{1 + b \theta_x}{\theta_x (Y q - b) - 1}
\]


Why?

1) “Self Control”: As the influent conc. increases, so does the conc. of active organisms in the reactor (eq 3.25 & 5.43).

The increased biomass is sufficient to consume the additional substrate that is added to the reactor.

\[
X_a = \frac{\theta_x}{\theta} \frac{Y (S^0 - S)}{1 + b \theta_x}
\]

[5.43]
5.7.1 CSTR with Settling and Cell Recycling

2) The organisms’ growth rate and SRT are equal to the inverse of each other.

\[ \theta_x = \frac{\text{active biomass in the system}}{\text{production rate of active biomass}} = \mu^{-1} \quad [3.22] \]

\[ \mu = Y \frac{q S}{K + S} - b \quad [3.9] \]

Constant SRT ( \( \theta_x \) ) \rightarrow Constant specific growth rate ( \( \mu \) )

\rightarrow Constant substrate ( \( S \) )
5.10 Engineering Design of Reactors

\[ \theta^d_x = SF \left[ \theta^\min_x \right]_{\text{lim}} \]  \[5.60\]  (Christensen and McCarty, 1975)

\[ \theta^d_x : \text{a design } \theta_x \]

<table>
<thead>
<tr>
<th>Loading</th>
<th>Implied SF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>10–80</td>
</tr>
<tr>
<td>High Rate</td>
<td>3–10</td>
</tr>
<tr>
<td>Low Rate</td>
<td>&gt;80</td>
</tr>
</tbody>
</table>

- **Conventional activated sludge treatment plants**: medium sized treatment systems that are expected to operate reliably with fairly constant supervision by reasonably skilled operator.

- **High Rate**: Highly skilled operator or the removal efficiency and high reliability is not as critical.

- **Low Rate**: Extended aeration
  : Operator attention is quite limited
  : Operators are present for a very short period time.
  : “shopping center”, or “apartment complex”
5.10 Engineering Design of Reactors

Factors to consider when selecting SF:

- High SF increases the degree of reliability of operation, but gives higher construction cost.

\[
\theta_x = \frac{VX_a}{X_a^e Q^e + X_a^w Q^w}
\]  \[5.35\]

- Low SF requires more continuous supervision and operators with increased skill.

Table 5.4  Factors to consider when selecting a safety factor for a given design loading

| Expected temperature variations |
| Expected wastewater variations |
| Flow rate                      |
| Wastewater concentration       |
| Wastewater composition         |
| Possible presence of inhibitory materials |
| Level of operator skill        |
| Efficiency required            |
| Reliability required           |
| Potential penalties for noncompliance |
| Confidence in design coefficients |
5.10 Engineering Design of Reactors

- Higher SS \( X \) makes the reactor volume smaller and thus less expensive for a given \( \theta_x^d \). However, high SS may require larger settling tanks, because increased loads of SS to the settling reactor.

\[
\theta_x = \frac{V X_a}{X_a^e Q^e + X_a^w Q^w}
\]  
[5.35]

**Table 5.5**  Typical values for total suspended solids concentration \( X \) in aerobic suspended-growth reactors with settling and recycle

<table>
<thead>
<tr>
<th>Floc Type</th>
<th>( X ) (mgSS/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>1,500–3,000</td>
</tr>
<tr>
<td>Poor compaction or low recycle rate</td>
<td>500–1,500</td>
</tr>
<tr>
<td>Good compaction or high recycle rate</td>
<td>3,000–5,000</td>
</tr>
</tbody>
</table>