CHAPTER 3.

Microbial Kinetics
3. Microbial Kinetics

- Microorganisms fuel their lives by performing redox reactions that generate the energy and reducing power needed to maintain and construct themselves.

- Because redox reactions are nearly always very slow unless catalyzed, microorganisms produce enzyme catalysts that increase the kinetics of their redox reactions to exploit chemical resources in their environment.

- Engineers want to take advantage of these microbially catalyzed reactions because the chemical resources of the microorganisms usually are the pollutants that the engineers must control:

  Organics (BOD, e⁻ donor), NH₄⁺ (e⁻ donor, nutrient), NO₃⁻ (e⁻ acceptor, nutrient), PO₄³⁻ (nutrient), etc.
3. Microbial Kinetics

• Engineers who employ microorganisms for pollution control must recognize two interrelated principles (connections between the active biomass (the catalyst) and the primary substances):

First:
Active microorganisms catalyze the pollutant removing reactions. So the rate of pollutant removal depends on the concentration of the catalyst, or the active biomass.

Second:
The active biomass is grown and sustained through the utilization of its energy - and electron (e⁻) - generating primary substrates. So the rate of biomass production is proportional to the utilization rate of the primary substrates.
3.1 Basic Rate Expressions

• Mass – balance modeling is based on i) the active biomass and ii) the primary substrate that limits the growth rate of the biomass.

• In the vast majority of cases, the rate limiting substrate is e⁻ donor. So the term substrate now refers to the primary e⁻ donor substrate.

• Jacques Monod: Nobel prize winner, director of Pasteur Institute.

Monod equation: relationship most frequently used to represent bacterial growth kinetics developed in the 1940s. His original work related the specific growth rate of fast – growing bacteria to the concentration of a rate limiting, e⁻ donor substrate.
3.1 Monod equation

Monod eq : largely empirical, but wide spread applied for microbial systems

\[
\mu_{syn} = \left( \frac{1}{X_a} \frac{dX_a}{dt} \right)_{syn} \quad = \mu \frac{S}{K + S} \quad \text{[3.1]}
\]

\( \mu_{syn} = \) specific growth rate due to synthesis (T\(^{-1}\))

\( X_a = \) concentration of active biomass (M\(_x\)L\(^{-3}\))

\( t = \) Time (T)

\( S = \) concentration of the rate-limiting substrate (M\(_s\)L\(^{-3}\))

\( K = \) substrate concentration giving one-half the maximum rate (M\(_s\)L\(^{-3}\))
3.1 Monod equation

Compare eq. 3.1 with eq. 1.13

$$\mu_{syn} = \left( \frac{1}{X_a} \frac{dX}{dt} \right)_{syn} = \mu \frac{S}{K + S} \quad [3.1]$$

$$v = v_m \frac{S}{K_M + S} \quad [1.13]$$

$\rightarrow$ Michaelis-Menten equation:
- developed in 1913
- theory of enzyme action and kinetics

$v$ : Reaction velocity

$K_M$ : Substrate concentration giving one-half of the maximum velocity

• Microorganisms are “bags full of enzymes” so that it is not surprising that the growth rate of microorganisms (Monod eq.) is related to the reactions of the catalysts that mediate many reactions (Michaelis and Menten eq.)
3.1 Basic Rate Expressions

Endogenous decay:

1) Environmental engineers study more slowly growing bacteria that has an energy demand for maintenance.

2) Active biomass has an energy demand for maintenance of cell functions (motility, repair and re-synthesis, osmotic regulation, transport and heat loss).

3) Flow of energy and electrons are required to meet maintenance needs.

4) The cell oxidize themselves to meet maintenance-energy needs.

\[
\mu_{dec} = \left( \frac{1}{X_a} \frac{dX_a}{dt} \right)_{decay} = -b \quad [3.2]
\]

\[b = \text{endogenous-decay coefficient}(T^{-1})\]

\[\mu_{dec} = \text{specific growth rate due to decay}(T^{-1})\]
3.1 Endogenous Decay

\[ \mu_{dec} = \left( \frac{1}{X_a} \frac{dX_a}{dt} \right)_{\text{decay}} = -b \]  

**Oxidation decay rate**: Although most of the decayed biomass is oxidized, a small fraction accumulates as inert biomass

\[ \mu_{resp} = \left( \frac{1}{X_a} \frac{dX_a}{dt} \right)_{\text{decay}} = -f_d b \]  

\[ f_d \text{ : fraction of active biomass that is biodegradable} \]

**The rate at which active biomass is converted to inert biomass**

\[ \mu_{inert} = \mu_{dec} - \mu_{resp} \]

\[ - \frac{1}{X_a} \frac{dX_i}{dt} = \left( \frac{1}{X_a} \frac{dX_a}{dt} \right)_{\text{inert}} = -b - (-f_d b) = -(1 - f_d) b \]

\[ X_i \text{ : inert biomass concentration}(M_xL^{-3}) \]

- inert biomass formed from active biomass decay
- inert biomass represents a fraction of only the organic portion of active biomass
3.1 Net specific growth rate

The net specific growth rate of active biomass ($\mu$) is the sum of new growth and decay

$$
\mu = \frac{1}{X_a} \frac{dX_a}{dt} = \mu_{syn} + \mu_{dec} = \mu \frac{\wedge S}{K + S} - b \quad [3.5]
$$
3.1 Net specific growth rate

If $S = 0$, then

$$\mu = \mu_{\text{syn}} + \mu_{\text{dec}} = \mu \frac{S}{K+S} - b$$

$$= 0 - b$$

At $S = 20K$, $\mu_{\text{syn}} = 0.95 \; \hat{\mu}$
3.1 Rate of substrate utilization

• The ultimate interest is to remove substrate

\[ r_{ut} = -\frac{q S}{K + S} X_a \]  [3.6]

\[ r_{ut} = \text{rate of substrate utilization (M}_s\text{L}^{-3}\text{T}^{-1}) \]
\[ q = \text{maximum specific rate of substrate utilization (M}_s\text{M}_x^{-1}\text{T}^{-1}) \]

• Substrate utilization and biomass growth are connected by

\[ \mu = q Y \]  [3.7]
3.1 Net rate of active-biomass growth

\[ \mu = q Y \]  \quad \text{[3.7]}  

\[ (T^{-1}) = (M_s M_x^{-1} T^{-1}) (M_x M_s^{-1}) = (T^{-1}) \]

\[ \rightarrow \text{The net rate of cell growth :} \]

\[ \mu = \mu \frac{S}{K + S} - b \]  \quad \text{[3.5]}  

\[ r_{net} = \mu X_a = \mu \frac{S}{K + S} X_a - bX_a \]

\[ r_{net} = Y \frac{q S}{K + S} X_a - bX_a \]  \quad \text{[3.8]}  

\[ r_{net} = \text{the net rate of active-biomass growth (M}_xL^{-3}T^{-1}) \]
3.1 Basic Rate Expressions

- \( r_{\text{net}} \) is connected with \( \mu \).

\[
\mu = \frac{r_{\text{net}}}{X_a} = Y \frac{qS}{K + S} - b \quad \text{[3.9]}
\]

Some prefer to think of cell maintenance as being a shunting of substrate-derived electrons and energy directly for maintenance. This is expressed by [3.10].

\[
\mu = Y \left( \frac{qS}{K + S} - m \right) \quad \text{[3.10]}
\]

\( m = \text{maintenance-utilization rate of substrate (} M_s M_x^{-1} T^{-1} \text{)} \)

When systems go to steady state, there is no difference in the two approach, and \( b = Ym \)
### 3.1 Basic Rate Expressions

\[ \mu = \frac{r_{\text{net}}}{X_a} = Y\left(\frac{qS}{K+S}\right) - b \]  

\[ \mu = Y\left(\frac{qS}{K+S} - m\right) = Y\left(-\frac{dS/dt}{X_a} m\right) \]  

\[ m = \text{maintenance-utilization rate of substrate (} M_sM_x^{-1}\text{ T}^{-1}) = \frac{b}{y} \]

At starvation state, the substrate utilization rate is less than \( m \)

\[ \frac{-dS/dt}{X_a} < m = \frac{b}{Y} \]

\[ \mu < 0 \]

At steady state, the net specific growth rate of cell (\( \mu \)), or the net yield (\( Y_n \)) is zero.

\[ \mu = 0 \quad , \quad X_a = \text{const.} \]

\[ \frac{-dS/dt}{X_a} = m = \frac{b}{Y} \]

- Any growth of biomass (\( mY \)) only supplements the decay of biomass (\( b \)).
  So apparently the concentration of biomass does not change.

- The energy supplied through substrate utilization is just equal to \( m \), the maintenance energy...
### 3.2 PARAMETER VALUES

**Table 3.1** Typical \( f_5^0 \), \( Y \), \( q \), and \( \hat{\mu} \) values for key bacterial types in environmental biotechnology

<table>
<thead>
<tr>
<th>Organism Type</th>
<th>Electron Donor</th>
<th>Electron Acceptors</th>
<th>C-Source</th>
<th>( f_5^0 )</th>
<th>( Y )</th>
<th>( q )</th>
<th>( \hat{\mu} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerobic, Heterotrophs</td>
<td>Carbohydrate BOD</td>
<td>O(_2)</td>
<td>BOD</td>
<td>0.7</td>
<td>0.49 gVSS/gBOD(_L)</td>
<td>27 gBOD(_L)/gVSS-d</td>
<td>13.2</td>
</tr>
<tr>
<td>Other BOD</td>
<td>O(_2)</td>
<td>BOD</td>
<td>0.6</td>
<td>0.42 gVSS/gBOD(_L)</td>
<td>20 gBOD(_L)/gVSS-d</td>
<td>8.4</td>
<td></td>
</tr>
<tr>
<td>Denitrifiers</td>
<td>BOD</td>
<td>NO(_3^-)</td>
<td>BOD</td>
<td>0.5</td>
<td>0.25 gVSS/gBOD(_L)</td>
<td>16 gBOD(_L)/gVSS-d</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>H(_2)</td>
<td>NO(_3^-)</td>
<td>CO(_2)</td>
<td>0.2</td>
<td>0.81 gVSS/gH(_2)</td>
<td>1.25 gH(_2)/gVSS-d</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>S(s)</td>
<td>NO(_3^-)</td>
<td>CO(_2)</td>
<td>0.2</td>
<td>0.15 gVSS/gS</td>
<td>6.7 gS/gVSS-d</td>
<td>1</td>
</tr>
<tr>
<td>Nitrifying Autotrophs</td>
<td>NH(_4^+)</td>
<td>O(_2)</td>
<td>CO(_2)</td>
<td>0.14</td>
<td>0.34 gVSS/gNH(_4^+)-N</td>
<td>2.7 gNH(_4^+)-N/gVSS-d</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>NO(_2^-)</td>
<td>O(_2)</td>
<td>CO(_2)</td>
<td>0.10</td>
<td>0.08 gVSS/gNO(_2^-)-N</td>
<td>7.8 gNO(_2^-)-N/gVSS-d</td>
<td>0.62</td>
</tr>
<tr>
<td>Methanogens</td>
<td>acetate BOD</td>
<td>acetate</td>
<td>acetate</td>
<td>0.05</td>
<td>0.035 gVSS/gBOD(_L)</td>
<td>8.4 gBOD(_L)/gVSS-d</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>H(_2)</td>
<td>CO(_2)</td>
<td>0.08</td>
<td>0.45 gVSS/gH(_2)</td>
<td>1.1 gH(_2)/gVSS-d</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Sulfide Oxidizing</td>
<td>H(_2)S</td>
<td>O(_2)</td>
<td>CO(_2)</td>
<td>0.2</td>
<td>0.28 gVSS/gH(_2)S-S</td>
<td>5 gS/gVSS-d</td>
<td>1.4</td>
</tr>
<tr>
<td>Autotrophs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sulfate Reducers</td>
<td>H(_2)</td>
<td>SO(_4^{2-})</td>
<td>CO(_2)</td>
<td>0.05</td>
<td>0.28 gVSS/gH(_2)</td>
<td>1.05 gH(_2)/gVSS-d</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>acetate BOD</td>
<td>SO(_4^{2-})</td>
<td>acetate</td>
<td>0.08</td>
<td>0.057 gVSS/gBOD(_L)</td>
<td>8.7 gBOD(_L)/gVSS-d</td>
<td>0.5</td>
</tr>
<tr>
<td>Fermenters</td>
<td>sugar BOD</td>
<td>sugars</td>
<td>0.18</td>
<td>0.13 gVSS/gBOD(_L)</td>
<td>9.8 gBOD(_L)/gVSS-d</td>
<td>1.2</td>
<td></td>
</tr>
</tbody>
</table>

- \( Y \) is computed assuming a cellular VSS\(_a\) composition of C\(_5\)H\(_7\)O\(_2\)N, and NH\(_4^+\) is the N source, except when NO\(_3^-\) is the electron acceptor; then NO\(_3^-\) is the N source. The typical units on \( Y \) are presented.
- \( q \) is computed using \( q = 1e^- \) eq/gVSS\(_a\)-d.
- \( \hat{\mu} \) has units of d\(^{-1}\).
3.2 PARAMETER VALUES---Y

- **Aerobic heterotrophs:**

\[
Y = 0.6 \frac{e^- \text{ eq cells}}{e^- \text{ eq donor}} \cdot \frac{113 \text{ gVSS}}{20 e^- \text{ eq cells}} \cdot \frac{1 e^- \text{ eq donor}}{8 \text{ gBOD}_L}
\]

\[= 0.42 \text{ gVSS/gBOD}_L\]

- **Denitrifying heterotrophs:**

\[
Y = 0.5 \frac{e^- \text{ eq cells}}{e^- \text{ eq donor}} \cdot \frac{113 \text{ gVSS}}{28 e^- \text{ eq cells}} \cdot \frac{1 e^- \text{ eq donor}}{8 \text{ gBOD}_L}
\]

\[= 0.25 \text{ gVSS/gBOD}_L\]

- **H$_2$-Oxidizing sulfate reducers:**

\[
Y = 0.05 \frac{e^- \text{ eq cells}}{e^- \text{ eq donor}} \cdot \frac{113 \text{ gVSS}}{20 e^- \text{ eq cells}} \cdot \frac{2 e^- \text{ eq donor}}{2 \text{ gH}_2}
\]

\[= 0.28 \text{ gVSS/gH}_2\]
Table 2.4  Cell formation ($R_c$) and common electron acceptor half-reactions ($R_a$)

<table>
<thead>
<tr>
<th>Reaction Number</th>
<th>Half-reaction</th>
<th>$\Delta G^0' \text{ kJ/e}^-$</th>
<th>eq</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cell Synthesis Equations ($R_c$)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Ammonium as Nitrogen Source</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C-1</td>
<td>$\frac{1}{5} \text{CO}_2 + \frac{1}{20} \text{HCO}_3^- + \frac{1}{20} \text{NH}_4^+ + \text{H}^+ + e^- = \frac{1}{20} \text{C}_5\text{H}_7\text{O}_2\text{N} + \frac{9}{20} \text{H}_2\text{O}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Nitrate as Nitrogen Source</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C-2</td>
<td>$\frac{1}{28} \text{NO}_3^- + \frac{5}{28} \text{CO}_2 + \frac{29}{28} \text{H}^+ + e^- = \frac{1}{28} \text{C}_5\text{H}_7\text{O}_2\text{N} + \frac{11}{28} \text{H}_2\text{O}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Nitrite as Nitrogen Source</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C-3</td>
<td>$\frac{5}{26} \text{CO}_2 + \frac{1}{26} \text{NO}_2^- + \frac{27}{26} \text{H}^+ + e^- = \frac{1}{26} \text{C}_5\text{H}_7\text{O}_2\text{N} + \frac{10}{26} \text{H}_2\text{O}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Dinitrogen as Nitrogen Source</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C-4</td>
<td>$\frac{5}{23} \text{CO}_2 + \frac{1}{46} \text{N}_2 + \text{H}^+ + e^- = \frac{1}{23} \text{C}_5\text{H}_7\text{O}_2\text{N} + \frac{8}{23} \text{H}_2\text{O}$</td>
<td></td>
</tr>
<tr>
<td><strong>Common Electron-Acceptor Equations ($R_a$)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I-14</td>
<td>Oxygen</td>
<td>$\frac{1}{4} \text{O}_2 + \text{H}^+ + e^- = \frac{1}{2} \text{H}_2\text{O}$</td>
<td>$-78.72$</td>
</tr>
<tr>
<td>I-7</td>
<td>Nitrate</td>
<td>$\frac{1}{5} \text{NO}_3^- + \frac{6}{5} \text{H}^+ + e^- = \frac{1}{10} \text{N}_2 + \frac{3}{5} \text{H}_2\text{O}$</td>
<td>$-72.20$</td>
</tr>
<tr>
<td>I-9</td>
<td>Sulfate</td>
<td>$\frac{1}{8} \text{SO}_4^{2-} + \frac{19}{16} \text{H}^+ + e^- = \frac{1}{16} \text{H}_2\text{S} + \frac{1}{16} \text{HS}^- + \frac{1}{2} \text{H}_2\text{O}$</td>
<td>$20.85$</td>
</tr>
<tr>
<td>O-12</td>
<td>CO$_2$</td>
<td>$\frac{1}{8} \text{CO}_2 + \text{H}^+ + e^- = \frac{1}{8} \text{CH}_4 + \frac{1}{4} \text{H}_2\text{O}$</td>
<td>$23.53$</td>
</tr>
<tr>
<td>I-4</td>
<td>Iron (III)</td>
<td>Fe$^{3+} + e^- = \text{Fe}^{2+}$</td>
<td>$-74.27$</td>
</tr>
</tbody>
</table>
3.2 PARAMETER VALUES---Y

- Experimental estimation of \( Y \)

- A small inoculum is grown to exponential phase and harvested from batch growth. Measure the changes in biomass and substrate conc. from inoculum until the time of harvesting.

- The true yield is estimated from

\[
Y = -\frac{\Delta X}{\Delta S}
\]

- The batch technique is adequate for rapidly growing cells, but can create errors when the cells grow slowly so that biomass decay cannot be neglected.
3.2 PARAMETER VALUES -- q

\[ q \] is controlled largely by \( e^- \) flow to the electron acceptor. For 20 °C, the maximum flow to the energy reaction is about \( 1 \ e^- \ eq / gVSS - d \), where 1 g VSS represents 1g of biomass.

\[ q_e = about \ 1 \ e^- \ eq / VSS - d \ at \ 20^0 C \]

\[ q = q_e / f_e^0 \]  \[ [3.11] \]

For example (Table 3.1), if \( f_s^0 = 0.7 \), then \( f_e^0 = 0.3 \). So

\[ q = q_e / f_e^0 = 8 g \ BOD_L / VSS - d / 0.3 = 27 g \ BOD_L / gVSS - d \]

\[ q \] is temperature function

\[ q_T = q_{20} (1.07)^{(T-20)} \]  \[ [3.12] \]

\[ q_T = q_T^R (1.07)^{(T-T^R)} \]  \[ [3.13] \]
3.2 PARAMETER VALUES--q

*b* is depends on species type and temperature.

\[ b = 0.1 - 0.3 \text{ /d for aerobic heterotrophs} \]

\[ b < 0.05 \text{ /d for slower-growing species} \]

\[ b_T = b_T^R (1.07)^{(T - T^R)} \]

Biodegradable fraction \((f_d)\) is quite reproducible and has a value near 0.8 for a wide range of microorganisms.
3.2 PARAMETER VALUES---k

\[ r_{net} = Y \frac{q S}{K + S} X_a - bX_a \]  \hspace{1cm} [3.8]

- **K** is the *Monod* half-maximum-rate concentration
  - most variable and least predictable parameter
  - Its value can be affected by the substrate`s affinity for transport or metabolic enzymes
  - mass-transport resistances (approaching of substrate and microorganisms to each other) ignored for suspended growth are lumped into the *Monod* kinetics by an increase in K
3.3 Basic Mass Balances

**Chemostat**: a completely mixed reactor, uniform and steady concentrations of active cell, substrate, inert biomass and any other constituents.

- **Active biomass**
  \[ 0 = \mu X_a V - Q X_a \]  
  \[ \text{[3.14]} \]

- **Substrate**
  \[ 0 = r_{ut} V + Q (S^0 - S) \]  
  \[ \text{[3.15]} \]

**Q** = constant feed flow rate

**S^0** = feed substrate concentration
3.3 Basic Mass Balances

- Active biomass

\[ 0 = \mu X_a V - QX_a \] \hspace{1cm} [3.14]

\[ \mu = \frac{r_{net}}{X_a} = Y \frac{\hat{q}S}{K + S} - b \] \hspace{1cm} [3.9]

\[ 0 = Y \frac{\hat{q}S}{K + S} X_a V - bX_a V - QX_a \] \hspace{1cm} [3.16]

\[ S = K \frac{1 + b \left( \frac{V}{Q} \right)}{Y\hat{q} \left( \frac{V}{Q} \right) - \left( 1 + b \left( \frac{V}{Q} \right) \right)} \] \hspace{1cm} [3.18]
3.3 Basic Mass Balances

• Substrate

\[ 0 = r_{ut} V + Q (S^0 - S) \]  \hspace{1cm} [3.15]

\[ r_{ut} = -\frac{\hat{q}S}{K + S} X_a \]  \hspace{1cm} [3.6]

\[ 0 = -\frac{\hat{q}S}{K + S} X_a V + Q (S^0 - S) \]  \hspace{1cm} [3.17]

\[ \frac{bX_a V + Q X_a}{Y} = \frac{\hat{q}S}{K + S} X_a V \]  \hspace{1cm} \text{from [3.16]}

\[ X_a = Y (S^0 - S) \frac{1}{1 + b \left( \frac{V}{Q} \right)} \]  \hspace{1cm} [3.19]
3.3 HRT & SRT

- **HRT (Hydraulic detention time)**

  \[
  \text{hydraulic detention time } (T) = \theta = V / Q \quad [3.20]
  \]

- **SRT (Solids retention time) = MCRT (Mean cell residence time)**
  
  = Sludge age

  \[
  \text{dilution rate } (T^{-1}) = D = Q / V \quad [3.21]
  \]

\[
\theta \chi = \frac{\text{active biomass in the system}}{\text{production rate of active biomass}} = \mu^{-1} \quad [3.22]
\]

\[
\mu = \frac{1}{X_a} \frac{dX_a}{dt} = \mu_{syn} + \mu_{dec} = \mu \frac{S}{K + S} - b \quad [3.5]
\]
3.3 SRT ($\theta_\chi$)

\[\theta_\chi = \frac{\text{active biomass in the system}}{\text{production rate of active biomass}} = \mu^{-1}\]  \[\text{[3.22]}\]

- **SRT**: Physiological status of the system
  : Information about the specific growth rate of the microorganisms
3.3 SRT \((\theta_x)\)

- SRT with quantitative parameters

\[ \theta_x = \frac{VX_a}{QX_a} = \frac{V}{Q} = \theta = \frac{1}{D} \] \[\text{[3.23]}\]

\[ \theta_x = \theta \quad \text{in chemostat at steady state} \]

\[ S = K \frac{1 + b \left( \frac{V}{Q} \right)}{Y \hat{q} \left( \frac{V}{Q} \right) - \left( 1 + b \left( \frac{V}{Q} \right) \right)} \] \[\text{[3.18]}\]

\[ S = K \frac{1 + b \theta_x}{Y \hat{q} \theta_x - \left( 1 + b \theta_x \right)} \] \[\text{[3.24]}\]

\[ \frac{V}{Q} = \theta = \theta_x \]

\[ X_a = Y \left( S^0 - S \right) \frac{1}{1 + b \left( \frac{V}{Q} \right)} \] \[\text{[3.19]}\]

\[ X_a = Y \left( \frac{S^0 - S}{1 + b \theta_x} \right) \] \[\text{[3.25]}\]
3.3 SRT ($\theta_\chi$)

- $S$ and $X_a$ are controlled by SRT($\theta_\chi$)

\[
S = K \frac{1 + b\theta_\chi}{Y\hat{q}\theta_\chi - (1 + b\theta_\chi)} \quad [3.24]
\]

\[
X_a = Y \left( \frac{S^0 - S}{1 + b\theta_\chi} \right) \quad [3.25]
\]

\[
[\theta_{\chi_{\text{min}}}^{\text{lim}}] = \frac{1}{Y\hat{q} - b}
\]

\[
S_{\text{min}} = K \frac{b}{Y\hat{q} - b}
\]
3.3 SRT \((\theta_X^{\min})\)

1) at very small \(\theta_X\) : \(S = S^0\), \(X_a = 0\)

\[ \theta_X^{\min} : \theta_X \] value at which washout begins

washout : \(\theta_X < \theta_X^{\min}\)

- by letting \(S = S^0\) in eq 3.24 and solving for \(\theta_X\)

\[
S = K \frac{1 + b \theta_X}{Yq \theta_X - (1 + b \theta_X)} \quad [3.24]
\]

\[
\theta_X^{\min} = \frac{K + S^0}{S^0(Yq - b) - bK} \quad [3.26]
\]

\(\theta_X = \theta = V/Q\), \(\theta_X = \uparrow\) then \(Q\) \(\downarrow\) (\(V = \text{const.}\))
3.3 SRT \((\theta^\text{min}_{\chi})_\text{lim}\)

\[
[\theta^\text{min}_{\chi}]_\text{lim} = \frac{1}{Y\hat{q} - b}
\]

\[
\theta^\text{min}_{\chi} = \frac{K + S^0}{S^0(Y\hat{q} - b) - bK}
\]  \[3.26\]

\(\theta^\text{min}_{\chi}\) decreases with increasing \(S^0\)

\[
[\theta^\text{min}_{\chi}]_\text{lim} = \lim_{s^0 \to \infty} \frac{K + S^0}{S^0(Y\hat{q} - b) - bK}
\]

\[
= \lim_{s^0 \to \infty} \frac{K / S^0 + 1}{(Y\hat{q} - b) - bK / S^0}
\]

\[
= \frac{1}{Y\hat{q} - b}
\]  \[3.27\]

\([\theta^\text{min}_{\chi}]_\text{lim}\) defines an absolute minimum \(\theta_{\chi}\) (or maximum \(\mu\)) boundary for having steady-state biomass.

It is a fundamental delimiter of a biological process.
3.3 SRT ( $\theta_{\chi}^{\min}$ )

2) For all $\theta_{\chi} > \theta_{\chi}^{\min}$, S declines monotonically with increasing $\theta_{\chi}$.

\[
[\theta_{\chi}^{\min}]_{\text{lim}} = \frac{1}{Y \dot{q} - b} \quad \theta_{\chi}^{\min}
\]

\[
\theta_{\chi} = \theta = \frac{V}{Q}, \quad \theta_{\chi} = \uparrow \text{ then } Q \downarrow (V = \text{const.})
\]
3.3 SRT ($S_{\min}$)

3) For very large $\theta_\chi$, $S$ approaches another key limiting value, $S_{\min}$.

$$S = K \frac{1 + b \theta_\chi}{Y \hat{q} \theta_\chi - (1 + b \theta_\chi)}$$

$$S_{\min} = \lim_{\theta_\chi \to \infty} K \frac{1 + b \theta_\chi}{Y \hat{q} \theta_\chi - (1 + b \theta_\chi)}$$

$$\theta_\chi = \theta = V/Q, \quad \theta_\chi \uparrow \text{ then } Q \downarrow (V = \text{const.})$$
• $S_{min}$ is the minimum concentration capable of supporting steady-state biomass.

- If $S < S_{min}$, the cells net specific growth rate is negative.

- In eq 3.9, $S \to 0$, then $\mu \to -b$.

$$\mu = \frac{r_{net}}{X_a} = Y \frac{qS}{K+S} - b \quad [3.9]$$

- Biomass will not accumulate or will gradually disappear.

- Therefore, steady-state biomass can be sustained only when $S > S_{min}$. 
3.3 SRT ($X_a$ variation)

4) When $\theta_\chi > \theta_\chi^{\text{min}}$, $X_a$ rises initially, because $Q(S^0 - S)$ increases as $\theta_\chi$ becomes larger.

- However, $X_a$ reaches a maximum and declines as decay becomes dominant ($Q(S^0 - S)$ decreases) for large $\theta_\chi$ (small $Q$).

- If $\theta_\chi$ were to extend to infinity, $X_a$ would approach zero (eq 3.25)

\[
[X_a^{\text{min}}]_{\text{lim}} = \frac{1}{Yq - b}
\]

\[
X_a = Y \left( \frac{S^0 - S}{1 + b\theta_\chi} \right)
\]  

\[ [3.25] \]
3.3 Microbiological Safety Factor

5) We can reduce $S$ from $S^o$ to $S_{min}$ as we increase $\theta \chi$ from $\theta_{\chi}^{min}$ to infinity.
   - The exact value of $\theta \chi$ we pick depends on a balancing of substrate removal ($S^o - S$), biomass production ($= Q X_a$), reactor volume ($V$) and other factors.
   - In practice, engineers often specify a microbiological safety factor, defined by $\frac{\theta \chi}{\theta_{\chi}^{min}}$.

- Typical safety factor = 5 ~ 100.
3.4 Mass Balances on Inert Biomass and Volatile Solids

- Because some fraction of newly synthesized biomass is refractory to self-oxidation, endogenous respiration leads to the accumulation of inactive biomass \((1 - f_d) bX_a V\).
- Real influents often contain refractory volatile suspended solids \(X_i^0\) that we cannot differentiate easily from inactive biomass.

**Steady-state mass balance on inert biomass**

\[
0 = (1 - f_d) bX_a V + Q (X_i^0 - X_i) \quad [3.29]
\]

- \(X_i\) = concentration of inert biomass
- \(X_i^0\) = input concentration of inert biomass

\[
\theta = \frac{V}{Q}
\]

\[
X_i = X_i^0 + X_a (1 - f_d) b \theta \quad [3.30]
\]
\[ X_i = X_i^0 + X_a (1 - f_d) b \theta \quad [3.30] \]

- Influent inert

- Inert biomass formed from active biomass decay

- Inert biomass represents a fraction of only the organic portion of active biomass

By introducing \( X_i^0 \), we are relaxing the original requirement that only substrate enters the influent.

\[
\text{From 3.25 and } \theta = \theta \chi, \quad X_a = Y \left( \frac{S^0 - S}{1 + b \theta \chi} \right) \quad [3.25]
\]

\[
X_i = X_i^0 + Y (S^0 - S) \frac{(1 - f_d) b \theta \chi}{1 + b \theta \chi}
\]
3.4 Maximum of inert biomass ($X_i$)

$$X_i = X_i^0 + Y(S^0 - S) \frac{(1 - f_d) b \theta \chi}{1 + b \theta \chi}$$

$$X_i^{\text{max}} = \lim_{\theta \chi \to \infty} X_i^0 + Y \left( \frac{S^0 - S}{1 + b \theta \chi} \right) (1 - f_d) \cdot b \cdot \theta \chi$$

$$= \lim_{\theta \chi \to \infty} X_i^0 + Y \left( \frac{S^0 - S}{1 + \frac{b}{b \cdot \theta \chi}} \right) (1 - f_d)$$

$$X_i^{\text{max}} = X_i^0 + Y \left( S^0 - S_{\text{min}} \right) (1 - f_d)$$

from Fig 3.3 $\lim_{\theta \chi \to \infty} S = S_{\text{min}}$
3.4 Mass Balances on inert Biomass and Volatile Solides

\[ X_{i_{\text{max}}} = X_{i_0} + Y(S^0 - S_{\text{min}})(1 - f_d) \]

- \( X_i \) increases monotonically from \( X_{i_0} \) to a maximum \( X_{i_{\text{max}}} \)
- Operation at a large \( \theta \) results in greater accumulation of inert biomass
3.4 \( X_v \): volatile suspended solids concentration (VSS)

\[
X_v = X_i + X_a
\]

\( X_v \): volatile suspended solids concentration (VSS)

\[
X_v = X_i^0 + X_a (1 - fd)b \theta_x + Xa \\
= X_i^0 + X_a (1 + (1 - fd)b \theta_x) \\
= X_i^0 + \frac{Y(S^0 - S) \cdot [1 + (1 - fd)b \theta_x]}{1 + b \theta_x}
\]

[3.31]
3.4 $X_v$: volatile suspended solids concentration (VSS)

- $X_v$ generally follow the trend of $X_a$, but it does not equal zero; 
  (When $X_a$ goes to zero, $X_v$ equals $X_i$)

- If $\theta_x < \theta_x^{\text{min}}$, $X_v = X_i^0$, $S = S^0$
3.4 \( Y_n \): net yield or observed yield (\( Y_{obs} \))

**from** \( [3.31] \)

\[
X_v = X_i^0 + X_a + X_a (1 - f_d) b \theta_x
\]

\[
= X_i^0 + Y(S^0 - S) \cdot \frac{[1 + (1 - f_d) b \theta_x]}{1 + b \theta_x}
\]

\[
= X_i^0 + Y_n (S^0 - S) \quad < \text{by intuition} >
\]

**Net accumulation of biomass from Synthesis and decay.**

\[
X_a = Y \left( \frac{S^0 - S}{1 + b \theta_x} \right) \quad [3.25]
\]

**\( Y_n \): net yield or observed yield (\( Y_{obs} \))**

\[
\therefore Y_n = Y \cdot \frac{1 + (1 - f_d) b \theta_x}{1 + b \theta_x} \quad [3.32]
\]

[3.32] is parallel to the relationship between \( f_s \) and \( f_s^0 \):

\[
\therefore f_s = f_s^0 \cdot \frac{1 + (1 - f_d) b \theta_x}{1 + b \theta_x} \quad [3.33]
\]
3.5 Soluble Microbial Products

SMP (soluble microbial products)

• Much of the soluble organic matter in the effluent from a biological reactor is of microbial origin.

• It is produced by the microorganisms as they degrade the organic substrate in the influent to the reactor.

• The major evidence for this phenomenon comes from experiments;

  “single soluble substrates of known composition were fed to microbial cultures and the resulting organic compounds in the effluent were examined for the presence of the influent substrate.”

  → “The bulk of the effluent organic matter was not the original substrate and was of high molecular weight, suggesting that it was of microbial origine.”

• It is called “Soluble Microbial Products (SMP)”.
3.5 Soluble Microbial Products

**SMP:** Biodegradable, although some at a very low rate

- Moderate formula weights (100s to 1000s)
- The majority of the effluent COD and BOD
- They can complex metals, foul membranes and cause color or foaming
- They are thought to arise from two processes:
  1) growth associated (UAP), 2) non-growth associated (BAP)

\[
\text{SMP} = \text{UAP} + \text{BAP}
\]

**UAP** (substrate-utilization-associated products)
- growth associated
- they are generated from substrate utilization and biomass growth
- they are not intermediates of catabolic pathways

**BAP** (biomass-associated products)
- non-growth associated
- related to decay and lysis of cell
- release of soluble cellular constituents through lysis and solubilization of particulate cellular components
3.5 SMP vs. EPS

- **SMP School** focuses on i) SMP, ii) active biomass and iii) inert biomass, but does not include EPS.

- **EPS School** focuses on i) EPS (soluble and bound) and ii) active biomass, but does not include SMP.

- Compared to other aspects of biochemical operations, little research has been done on the production and fate of soluble microbial products. So little is known about the characteristics of **UAP** and **BAP**.
3.5 UAP and BAP formation kinetics

- **UAP formation kinetics** :

  \[ r_{UAP} = -k_1 r_{ut} \]

  \[ r_{UAP} = \text{rate of UAP-formation} \ (M_p L^{-3} T^{-1}) \]

  \[ k_1 = \text{UAP- formation coefficient} \ (M_p M_s^{-1}) \]

- **BAP formation kinetics** :

  \[ r_{BAP} = k_2 X_a \]

  \[ r_{BAP} = \text{rate of BAP-formation} \ (M_p L^{-3} T^{-1}) \]

  \[ k_2 = \text{BAP- formation coefficient} \ (M_p M_x^{-1}) \]
3.5 Soluble Microbial Products

Example 3.3 Effluent $BOD_5$

The BOD exertion equation:

$$BOD_t = BOD_L (1 - \exp\{-kt\}) \quad [3.41]$$

$$= BOD_L - BOD_L \exp\{-kt\}$$

$$BOD_5 : BOD_L = 1 - \exp\{-k \cdot 5d\} \quad [3.42]$$

<table>
<thead>
<tr>
<th></th>
<th>$k$</th>
<th>$BOD_5:BOD_L$ ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>original substrate</td>
<td>0.23/d</td>
<td>0.68</td>
</tr>
<tr>
<td>active biomass</td>
<td>0.1/d</td>
<td>0.40</td>
</tr>
<tr>
<td>SMP</td>
<td>0.03/d</td>
<td>0.14</td>
</tr>
</tbody>
</table>
3.6 Nutrients and Electron Acceptors

• Biotechnological Process must provide sufficient **nutrients and electron acceptors** to support biomass growth and energy generation.

• Nutrients, being elements comprising the physical structure of the cells, are needed *in proportion to the net production of biomass*.

• Active and inert biomass contain nutrients, as long as they are produced microbiologically.

• **e⁻ acceptor** is consumed *in proportion to e⁻ donor utilization* multiplied by the sum of *exogenous and endogenous flows of e⁻* to the terminal acceptor.
3.6 How to determine nutrient requirements

- Nutrients are needed in proportion to the net production of biomass \( r_{ut} Y_n \)
- In chemostat model, rate of nutrients consumption \( (r_n) \)

\[
r_n = \gamma_n r_{ut} Y_n \quad Y = (M_x M_s^{-1})
= \gamma_n r_{ut} Y \frac{1 + (1 - f_d) b \theta_x}{1 + b \theta_x} \quad [3.43]
\]

\( r_n = \) rate of nutrient consumption \( (M_n L^{-3} T^{-1}) \) (negative value)

\( \gamma_n = \) the stochiometric ratio of nutrient mass to VSS for the biomass \( [M_n M_x^{-1}] \)

\( \gamma_N = 14 \text{g N/113g cell VSS} = 0.124 \text{g N/g VSS} \quad < C_5H_7O_2N > \)

\( \gamma_P = 0.2 \times \gamma_N = 0.025 \text{ g P/g VSS} \)

\[
[ \quad Y_n = Y \cdot \frac{1 + (1 - f_d) b \theta_x}{1 + b \theta_x} \quad ] \quad [3.32]
\]
3.6 Nutrient requirements

- Overall mass balance

\[ 0 = Q C_n^0 - Q C_n + r_n V \quad [3.44] \]

\[ C_n = C_n^0 + r_n \theta \quad [3.45] \]

\( C_n^0 \): Influent concentration of nutrient (M\(_n\) L\(^{-3}\))

\( C_n \): Effluent concentration of nutrient (M\(_n\) L\(^{-3}\))

\( r_n \): rate of nutrient consumption (M\(_n\) L\(^{-3}\)T\(^{-1}\)) (negative value)

* The supply of nutrients must be supplemented if \( C_n \) is negative in eq 3.45.
3.6 Use rate of Electron Acceptors

- Use rate of electron acceptor \( \Delta S_a / \Delta t \ (M_a T^{-1}) \)

\[
OD \ inputs = QS^0 + 1.42 \frac{gCOD}{gVSS} X_v^0 Q \quad [3.46]
\]

\[
OD \ outputs = QS + Q(SMP) + 1.42 \frac{gCOD}{gVSS} X_v Q \quad [3.47]
\]

- The acceptor consumption as \( O_2 \) equivalents, \( \Delta S_a / \Delta t = OD \ inputs - OD \ outputs \)

\[
\frac{\Delta S_a}{\Delta t} = \gamma_a Q \left[ S^0 - S - SMP + 1.42(X_v^0 - X_v) \right] \quad [3.48]
\]

\( \gamma_a = 1 \text{ g } O_2 / \text{ g COD} \) for oxygen

\( \gamma_a = 0.35 \text{ g } NO_3^- - \text{ N/g COD} \) for \( NO_3^- - \text{ N} \)

\[
\begin{align*}
O_2 \quad (32 \text{ g}) + \quad 4 \text{ e}^- & \rightarrow \quad 2 \text{ O}^2^- \\
NO_3^- \quad (14 \text{ g}) + \quad 5 \text{ e}^- & \rightarrow \quad \text{ N} \quad 0
\end{align*}
\]

\[
14 \text{ g } NO_3^- - \text{ N} / 5 \text{ e}^- / 32 \text{ g } O_2 / 4 \text{ e}^- = 0.35 \text{ g } NO_3^- - \text{ N } / \text{ g } O_2
\]
3.6 Amount of $e^-$ acceptors to be supplied

- the acceptor can be supplied in the influent flow or by other means, such as aeration to provide oxygen ($R_a$).

\[
\frac{\Delta S_a}{\Delta t} = Q\left[S_a^0 - S_a\right] + R_a \quad [3.49]
\]

$\Delta S_a$: Amount of $e^-$ acceptors to be supplied

$\text{Acceptor consumption rate}$

$Ra$: the required mass rate of acceptor supply ($M_a T^{-1}$)

$S_a^0$: Influent concentration of $e^-$ acceptor ($M_a L^{-3}$)

$S_a$: Effluent concentration of $e^-$ acceptor ($M_a L^{-3}$)
3.7 Input Active Biomass

- Three circumstances for inputs of active biomass in degradation of the substrate:
  - When processes are operated in series, the downstream process often receives biomass from the upstream process.
  - Microorganisms may be discharged in a waste stream or grown in the sewers.
  - Bioaugmentation is the deliberate addition of microorganisms to improve performance.

- When active biomass is input, the steady-state mass balance for active biomass (3.16), must be modified.

\[
0 = \mu X_a V - QX_a \quad [3.14]
\]
\[
0 = Y \frac{\hat{q} S}{K + S} X_a V - bX_a V - QX_a \quad [3.16]
\]
\[
0 = QX_a^0 - QX_a + Y \frac{\hat{q} S}{K + S} X_a V - bX_a V \quad [3.50]
\]
3.7 Input Active Biomass

\[ 0 = QX_a^0 - QX_a + Y \frac{\hat{q}S}{K + S} X_a V - bX_a V \quad [3.50] \]

- Eq 3.50 can be solved for \( S \) when active biomass is input,

\[ S = K \frac{1 + b \theta_x}{Y \hat{q} \theta_x - (1 + b \theta_x)} \quad [3.51] \]

- The solution for \( s \) (eq 3.51) is exactly the same as eq 3.24,

\[ S = K \frac{1 + b \theta_x}{Y \hat{q} \theta_x - (1 + b \theta_x)} \quad [3.24] \]

but the \( \theta_x \) is now redefined.

- When the SRT is redefined as the reciprocal of the net specific growth rate,

\[ \theta_x = \mu^{-1} = \frac{X_a V}{QX_a - QX_a^0} \quad [3.52] \]
3.7 **HOW does** $X_a^0$ **affect** $S$ and washout?

- $X_a^0 = 0$, washout occurs for $\theta_x$ of about 0.6 d.
- As $X_a^0$ increases, complete washout is eliminated, because the reactor always contains some biomass.
- Increasing $X_a^0$, also makes $S$ lower, and the effect is most dramatic near washout.

**Figure 3.5** Effect of influent active biomass and $\theta_x$ on effluent substrate concentration for a chemostat at steady state. The parameters used are $Y = 0.44$ mg VSS$_a$/mg, $\dot{q} = 5$ mg/mg VSS$_a$$\cdot$d, $K = 20$ mg/l, and $b = 0.2$/d.
3.7 Input Active Biomass

For biomass,  
\[ 0 = QX_a^0 - QX_a + Y \frac{\hat{q}S}{K + S} X_a V - bX_a V \]  \[ 3.50 \]

For substrate,  
\[ 0 = -\frac{\hat{q}S}{K + S} X_a V + Q(S^0 - S) \]  \[ 3.17 \]

For inert,  
\[ 0 = (1 - f_d) bX_a V + Q(X_i^0 - X_i) \]  \[ 3.29 \]

\[ X_v = X_i^0 + X_a (1 + (1 - f_d)b\theta_x) \]

- Due to the change in definition of $\theta_x$, solutions of the mass balances differ.

\[ X_a = \frac{\theta_x}{\theta} \left[ Y(S^0 - S) \frac{1}{1 + b\theta_x} \right] \]  \[ 3.53 \]

\[ X_i = X_i^0 + X_a (1 - f_d)bX_a \theta \]  \[ 3.54 \]

\[ X_v = X_i + X_a = X_i^0 + X_a (1 + (1 - f_d)b\theta_x) \]  \[ 3.55 \]

When $X_a^0 = 0$, \[ X_v = X_i^0 + X_a (1 + (1 - f_d)b\theta_x) \]  \[ 3.31 \]


3.7 Input Active Biomass

- Due to the change in definition of $\theta x$, solutions of the mass balances differ.

$$X_a = \frac{\theta x}{\theta} \left[ Y(S^0 - S) \frac{1}{1 + b \theta x} \right]$$ \[3.53\]

- $\theta x/\theta$ ratio represents the build up of active biomass by the input.

Rearranging Eq. 3.52,

$$\frac{\theta x}{\theta} = \frac{1}{1 - X^0_a / X_a}$$ \[3.56\]

- $X_a^0 < X_a$: $\theta x > \theta$
- $X_a^0 > X_a$: $\theta x$ is negative.

The net biomass decay which makes the steady-state $\mu$ negative and sustain $S < S_{\min}$. If $\theta x$ is negative, $S < S_{\min}$ from eqs below

$$S = K \frac{1 + b \theta x}{Y\hat{q} \theta x - (1 + b \theta x)}$$

$$S_{\min} = \lim_{\theta x \to \infty} K \frac{1/\theta x + b}{Y\hat{q} - 1/\theta x - b}$$

$$= K \frac{1/\theta x + b}{Y\hat{q} - 1/\theta x - b}$$

$$= K \frac{b}{Y\hat{q} - b}$$
3.8 Hydrolysis of Particulate and Polymeric Substrates

• Organic substrate of particles (large polymers)
  - Organic substrates that originate as particles or large polymer take important application in environmental biotechnology

• Effect of hydrolysis
  - Before the bacteria can begin the oxidation reaction, the particles or large polymers must be hydrolyzed to smaller molecules
  - Extracellular enzymes catalyze these hydrolysis reaction
  - Hydrolysis kinetics is not settled, because the hydrolytic enzymes are not necessarily associated with or proportional to the active biomass
  - Level of hydrolytic enzymes is not established, and measurement is neither simple
3.8 Hydrolysis of Particulate and Polymeric Substrates

- Hydrolysis of kinetics

  - Reliable approach is a first-order relationship

\[ r_{hyd} = -k_{hyd} S_p \] \[ \text{[3.57]} \]

  * \( r_{hyd} \): rate of accumulation of particulate substrate due to hydrolysis \((M_sL^{-3}T^{-1})\)
  * \( S_p \): concentration of the particulate (or large-polymer) substrate \((M_sL^{-3})\)
  * \( k_{hyd} \): first-order hydrolysis rate coefficient \((T^{-1})\)

- \( k_{hyd} \) is proportional to the concentration of hydrolytic enzymes as well as to the intrinsic hydrolysis kinetics of the enzymes

- some researchers include the active biomass concentration as part of \( k_{hyd} \)

\[ k_{hyd} = k_{hyd}' X_a \quad (k_{hyd}' = \text{specific hydrolysis rate coefficient}) \]
- When Equation 3.57 is used, the steady-state mass balance on particulate substrate

\[ 0 = r_{ut} V + Q(S^0 - S) \quad [3.14] \]

\[ 0 = Q(S_p^0 - P_p) - k_{hyd} S_p V \quad [3.58] \]

\[ S_p = \frac{S_p^0}{1 + k_{hyd} \theta} \quad [3.59] \]

- \( \theta \) represent the liquid detention time and should not be substituted by \( \theta_x \)
3.8 Hydrolysis of Particulate and Polymeric Substrates

• The steady-state mass balance on soluble substrate

- The steady-state mass balance on substrate

\[
0 = -\frac{\hat{q}S}{K + S} X_a V + Q(S^0 - S) \quad [3.17]
\]

- The steady-state mass balance on soluble substrate

\[
0 = (S^0 - S) - \frac{\hat{q}S}{K + S} X_a V / Q + k_{hyd} S_p V / Q \quad [3.60]
\]

* \(S^0\) effectively is increased by \(k_{hyd} S_p V / Q\)
Other constituents of particulate substrate also are conserved during hydrolysis.

- Good examples of particulate substrate: the nutrient nitrogen, phosphate, sulfur

\[ r_{\text{hydn}} = \gamma_n k_{\text{hyd}} S_p \]  \[3.61\]

* \( r_{\text{hydn}} \): rate of accumulation of a soluble form of nutrient \( n \) by hydrolysis (\( M_n \text{L}^{-3} \text{T}^{-1} \))

* \( \gamma_n \): stoichiometric ratio of nutrient \( n \) in the particulate substrate (\( M_n M_s^{-1} \))
3.8 Hydrolysis of Particulate and Polymeric Substrates

Example 3.6

**EFFECT OF HYDROLYSIS**  Example 3.1 showed that a chemostat fed 500 mg BOD\(_L\)/l with a liquid detention time of 2 d produced an effluent quality of:

\[
S = 1.7 \text{ mg BOD}_L/\text{l} \\
X_a = 161 \text{ mg VSS}_a/\text{l} \\
X_v = 221 \text{ mg VSS}/\text{l} \\
\text{SMP} = 32 \text{ mg COD}_p/\text{l} \\
\text{Soluble BOD}_L = 33.5 \text{ mg BOD}_L/\text{l} \\
\text{Total COD} = 348 \text{ mg COD}/\text{l}
\]

We consider now that the influent also contains biodegradable particulate organic matter with a concentration of 100 mg COD/l, and the hydrolysis rate coefficient is \( k_{hyd} = 0.2/\text{d} \).

The computations to predict the new effluent quality proceed with the following steps:

1. \( S_p \) is computed from Equation 3.59:
   \[
   S_p = \frac{100 \text{ mg COD}/\text{l}}{1 + (0.2/\text{d})(2 \text{ d})} \\
   = 71 \text{ mg COD}/\text{l}
   \]

2. Since \( \theta_x = \theta \) remains 2 d, and no active biomass is input, \( S = 1.7 \text{ mg BOD}_L/\text{l} \).

3. The effluent and reactor biomass concentrations are determined in parallel to example 3.1, except that the effective \( S^0 \) is now:
   \[
   S^0 = 500 \text{ mg/l} + (100 - 71)\text{mg/l} \\
   = 529 \text{ mg BOD}_L/\text{l}
   \]
3.8 Hydrolysis of Particulate and Polymeric Substrates

Y of casein (pp. 170)

\[
X_a = 0.42 \frac{g \text{ VSS}_a}{g \text{ BOD}_L} \left( 529 - 1.7 \frac{\text{mg BOD}_L}{1} \right) \frac{1}{1.3} \\
= 170 \text{ mg VSS}_a/l
\]

\[
X_i = 50 \text{ mg VSS}_i/l + 170 \frac{\text{mg VSS}_a}{1} (1 - 0.8)(0.15/d)(2 \text{ d}) \\
= 60 \text{ mg VSS}_i/l;
\]

\[
X_v = X_a + X_i + S_p = 170 + 60 + \frac{71}{1.42} = 280 \text{ mg VSS}/l
\]

\[
X_a = \left[ Y(S^0 - S) \frac{1}{1 + b \theta_x} \right] \frac{\theta_x}{\theta} \quad [3.53]
\]

\[
X_i = X_i^0 + (1 - f_d) b X_a \theta \quad [3.54]
\]

\[
X_v = X_i + X_a \quad [3.55] \rightarrow X_v = X_i + X_a + S_p \quad [3.55']
\]

Thus, the amount of active biomass is augmented by the hydrolysis of particulate COD, while the VSS also is augmented by the remaining biodegradable particulate COD.

4. The detailed computations for SMP are omitted, as they are exactly analogous to Example 3.1. The result is

UAP = 9.4 mg COD$_p$/l
BAP = 23.2 mg COD$_p$/l
SMP = 32.6 mg COD$_p$/l

5. The effluent concentrations of COD and BOD$_L$ are affected by the changes in $X_a$, $X_i$ and SMP, as well as by the residual particulate organic substrate $S_p$. All increase.

Soluble COD and BOD$_L$ = $S + $SMP

= 1.7 + 32.7

= 34.4 mg COD/l

COD’/weight = the conversion factor (pp. 129, 181)

Total COD

= $S + $SMP + 1.42 X_v

= 1.7 + 32.7 + 1.42 \cdot 280

= 432 mg COD/l

The biodegradable fraction = 0.8 (pp. 171)

Total BOD$_L$

= $S + $SMP + f_d X_a + S_p

= 1.7 + 32.7 + 1.42 \cdot 0.8 \cdot 170 + 71

= 299 mg BOD$_L$/l